Robust surface reconstruction from gradient fields

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Surface reconstruction from gradient fields is a fundamental problem to shape from shading and photometric stereo. Proposed is a surface reconstruction method that is robust to both noise and outliers. The reconstruction problem is formulated to linear decoding in compressed sensing, by assuming the outliers are sparsely distributed. A Laplacian term is additionally employed to increase information in the construction matrix and suppress noise and/or outliers. Experimental results validate that the proposed method significantly outperforms the state of the art, and can produce satisfactory reconstruction even in the very extreme situation of 60% outliers.

Introduction: Shape from gradient fields has a long research history in computer vision, and is the final step for many applications involving gradient manipulation and estimation. When the input data is corrupted by noise and outliers, it could not be directly integrated. The traditional methods solve the reconstruction problem under the least squares sense, and are suitable to Gaussian noise polluted data. To handle outliers, Reddy *et al.* [1] proposed a confining method that detects the sparse errors produced by a curl operator in advance, and then integrates using least squares. This method stops the influence of outliers from spreading but could not annihilate them completely.

Recent work [2] shows that compressed sensing is powerful in removing sparsely distributed salt-andpepper noise from images, using basis pursuit [3]. Candes and Tao [4] extend the basis pursuit by introducing a linear decoding strategy to handle the linear error correcting problem with ℓ_1 minimisation. The observed codeword is generated by multiplying a construction matrix to an input message. It is proved that the message can be exactly recovered even when the codeword is corrupted by a large number of outliers, providing that the construction matrix satisfies the adjoint uniform uncertainty principle (UUP).

In this Letter, the surface integration is formulated to the error correcting problem, and the corrupted surface can be recovered using ℓ_1 minimisation. By further adding a Laplacian term in the construction matrix, we increase the information for decoding and suppress noise simultaneously. The proposed method can reconstruct surfaces robustly in the existence of severe outlier corruption, even when the adjoint UUP is not satisfied.

Proposed method: Measured surface is usually organised in the range-image form, which can be written in terms of depth function $z = f(x, y)$, where x and y are Cartesian co-ordinates. The gradient fields of $f(x, y)$ are given by finite difference form as

$$z_x(x, y) = \frac{z(x+1, y) - z(x-1, y)}{2}$$
$$z_y(x, y) = \frac{z(x, y+1) - z(x, y-1)}{2} \tag{1}$$

which can be written in its matrix-vector notation as

$$\begin{pmatrix} \mathbf{z_x} \\ \mathbf{z_y} \end{pmatrix} = \mathbf{Dz} \tag{2}$$

where $\mathbf{z}$ denotes the stacked depth values in vector form, $\mathbf{z_x}$ and $\mathbf{z_y}$ are stacked gradient vectors, and $\mathbf{D}$ denotes the gradient operator matrix. Each row of $\mathbf{D}$ has two nonzeros entries, namely ± 0.5, in the positions corresponding to the particular difference operation. Equation (2) can be treated as a gradient linear encoding operation, which translates depth to gradient fields.

We additionally construct the popular Laplacian kernel for the surface as

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{pmatrix} \tag{3}$$

and consequently change the convolution operation to its matrix form

$$\text{Laplacian}(\mathbf{z}) = \mathbf{Lz} \tag{4}$$

where Laplacian$(\cdot)$ denotes finite difference Laplacian operator. Each row of $\mathbf{L}$ has five nonzero entries corresponding to the nonzero elements in the convolution kernel (3). We treat the minimisation of the surface Laplacian as a regularisation term and incorporate it in the objective function

$$\hat{\mathbf{z}} = \arg\min \|\mathbf{g^0} - \mathbf{Az}\|_1 \tag{5}$$

where $\hat{\mathbf{z}}$ denotes the solution of depths, $\mathbf{g}^0$ is the observed stacked gradient data incorporated with $\mathbf{0}$ vector (corresponding to the minimisation of surface Laplacian), and $\mathbf{A} = \begin{pmatrix} \mathbf{D} \\ \lambda\mathbf{L} \end{pmatrix}$ is the construction matrix, with λ being the weight of the regularisation term. The objective function (5) can be formulated as a linear programming problem, which can be effectively solved using convex optimisation [5].

Results: Experiments were conducted to evaluate Reddy's method [1] and the proposed method. In both methods, we all assume Neumann boundary conditions for integration. Our investigation indicates that the proposed method is insensitive to parameter λ and therefore we fixed $\lambda = 0.3$ hereafter. In the experiment, the noise was assumed to be additive Gaussian determined with variance σ^2. The outliers can be either additive or multiplicative. In the case of an additive outlier, the corrupted gradients become $z'_a = z_a + \{-10, 10\}$, where $a \in \{x, y\}$, and in the case of a multiplicative outlier, $z'_a = z_a\{-10, 10\}$. Fig. 1 shows the reconstruction results of *Peak* surface when the gradient fields are corrupted by multiplicative outliers. It is observed that Reddy's method can annihilate partial outliers and confine the influence of outliers in the case of 10% corruption, but completely fails in the case of 40% corruption. This is because the large corruption makes the curl operation in Reddy's method hard to detect all outliers in integration loops. In comparison, the proposed method removes all outliers and produces a perfect target surface when there are 10% outliers, and still performs quite satisfactorily when an outlier becomes more severe. Fig. 2 illustrates the root mean square error (RMSE) of the reconstructed surfaces with respect to different outlier percentages. As shown, in both cases of additive and multiplicative outliers, the errors of Reddy's method increase dramatically when the outlier percentage becomes large. Instead, the proposed method keeps high accuracy and is very stable, even when the gradient data contain 60% outliers. Fig. 3 shows the reconstruction results of *Mozart* and *Vase* surfaces when the gradients are simultaneously corrupted by Gaussian noise ($\sigma = 0.1$) and 20% additive outliers. Again, the proposed method produces satisfactory surface reconstruction, while Reddy's method fails.

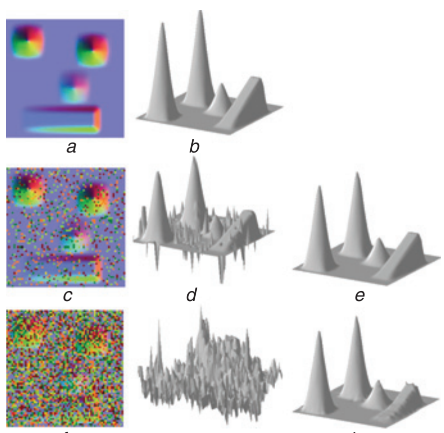

Fig. 1 *Reconstruction of Peak surface when gradient fields corrupted by multiplicative outliers*

a Ground truth normal map
b Ground truth depth
c Normal map with 10% positions corrupted
d Reconstruction of Fig. 1*c* by Reddy's method
e Reconstruction of Fig. 1*c* by proposed method
f Normal map with 40% positions corrupted
g Reconstruction of Fig. 1*f* by Reddy's method
h Reconstruction of Fig. 1*f* by proposed method

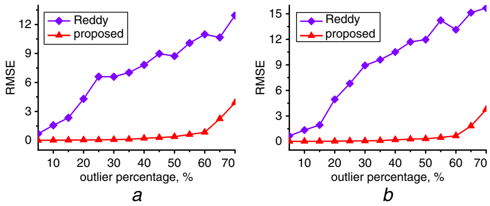

Fig. 2 *Reconstruction error with respect to outlier percentage for Peak surface*

a Additive outliers
b Multiplicative outliers

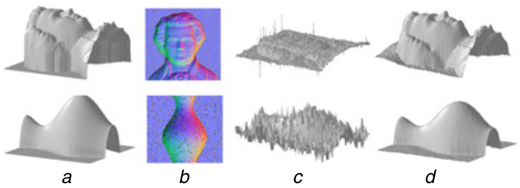

Fig. 3 *Reconstruction of Vase and Mozart surfaces when gradient fields corrupted by both noise and additive outliers*

a Ground truth depths
b Normal maps corrupted by both outliers (20% positions) and noise ($\sigma = 0.1$)
c Reddy's method
d Proposed method

Conclusion: We have proposed a robust method to recover depths from gradient fields. We first formulated surface reconstruction to the gradient decoding problem, and then employed a Laplacian term to increase the information in the construction matrix and suppress noise/outliers at the same time. The experimental results verified that, even when the outlier corruption is extremely severe, the proposed method can still produce satisfactory surface reconstruction.

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One or more of the Figures in this Letter are available in colour online.

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